PRACTICE PROBLEMS FOR FINAL EXAM

1. Find the power series represention of $f(x) = Arc\tan x$ in terms of x^n (i.e., find its Taylor series at $x = 0$). Determine of the interval of convergence.

2. Calculate the following limits if they exist

$$(i) \lim_{(x,y)\to(0,0)} \frac{x-y}{\sqrt{x^2+y^2}} \qquad (ii) \lim_{(x,y)=(0,0)} \frac{2x^2-3y^4}{5xy^2}.$$

3. Discuss the continuity of $f(x,y) = \begin{cases} \frac{x-y}{\sqrt{x^2+y^2}} & \text{if } (x,y) \neq (0,0) \\ 5 & \text{if } (x,y) = (0,0) \end{cases}$.

4. Find $\frac{\partial w}{\partial s}$ and $\frac{\partial w}{\partial t}$ for

$$w = xy^2z^3, \quad x = 3\sqrt{st}, \quad y = \sin 2t, \quad z = s + \ln(t+s)$$

5. Determine the local **maximum** / minumum / **saddle** points of

$$f(x,y) = 4x^3 + y^2 - 2x^2y + 1$$

6. Calculate the **directional derivative** of $h(x,y,z) = x^3 - xy^2 - z$ at $P(1,1,0)$ in the direction of $\overrightarrow{v} = 2i - 3j + 6k$

7. By using linearization (i.e., tangent plane approximation), calculate approximately the value of

$$e^{0.1}\ln(2.72) \approx$$

8. Find the maximum value of $f(x,y,z) = xyz$ under the condition $x + 2y + z = 2$ by using Lagrange Multipliers Method.

9. Calculate the volume of the half sphere with radius 5 cm by using the multiple integrals.

10. Change the order of integration in $\int_0^1 \int_{-2y}^{2y} f(x,y)dxdy$.

11. Calculate $\iint\limits_R \sin(x^2+y^2)dA$ where $R = \{(x,y)\,;\pi^2 \leq x^2+y^2 \leq 4\pi^2\}$.

12. Calculate $\int_{-1}^{1}\int_0^2\int_0^{x+z} x^2yz^2dydxdz$.