PRACTICE PROBLEMS FOR FINAL EXAM

1. Find all values of the constants m and b for which the function

$$f(x) = \begin{cases} \sin x & \text{if } x < \pi \\ mx + b & \text{if } x \geq \pi \end{cases}.$$

   a. is continuous at $x = \pi$.

   b. differentiable at $x = \pi$.

2. Calculate the following limits

$$\text{(a). } \lim_{x \to 0} \frac{\sin(101x)}{x+1-\cos(x)}, \quad \text{(b) } \lim_{x \to 0} \frac{\int_x^{x^2} \sin t \, dt}{x}, \quad \text{(c). } \lim_{x \to \infty} \frac{\sin(x)}{e^x}, \quad \text{(d). } \lim_{x \to \infty} \frac{e^x - e^{-101x}}{e^x + e^{-101x}}$$

3. Find the tangent line and normal line equations at $(1, -1)$ for the curve

$$x^3 - y\sin(x + y) = 1.$$

4. For the function below

$$y = (101^x + 1)^{1/x}$$

   a. Find $\lim_{x \to \infty} (101^x + 1)^{1/x}$

   b. $\frac{dy}{dx}(1) = ?$

5. Find the linearization of the function

$$G(x) = \sqrt{1 + 2x}$$

at the point $x = 4$. Calculate $\sqrt{8.96}$ aproximately by utilizing the linearization of G.

6. The area of a circular region is increasing at a rate of 96π square meters per second ($m^2/\sec$). When the area of region is 64π square meters, how fast is **the radius of the region** increasing in meters per second?

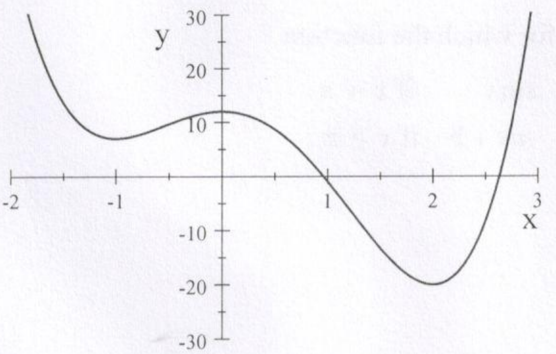
7. By utilizing the Mean Value Theorem, show that $x^5 + 15x + 1 = 0$ has **a unique** solution (i.e. zero).

8. Find the absoluate maximum and minumum values of the function $h(x) = 2\sin(x) - \sin^2(x)$ in $[0, 2\pi]$. Sketch the graph of $f(x)$ by following the steps below:

$$f(x) = \frac{x}{x + \pi}$$

   a. Find the domain, x –intercept and y –intercept.
   b. Find the asymptotes (if they exist!).
   c. Determine critical points, the intervals at which $f(x)$ increases and decreases, and local max/min values
   d. Determine the intervals at which $f(x)$ is concave up or down, and inflection points.
   e. Make a table consisting of all information above and then skech the graph of $f(x)$.

9. The graph of $y = f(x)$ is given below and $F(x) = \int_0^x f(t)dt$.



Find the critical points of F. Determine the intervals at which F is increasing, decreasing, concave up and concave down.

10. Sketch the graph of $g(x)$ by following the steps below:

$$g(x) = \frac{e^x}{x}$$

a. Find the domain, x −intercept and y −intercept.
b. Find the asymptotes (if they exist!).
c. Determine critical points, the intervals at which $g(x)$ increases and decreases, and local max/min values
d. Determine the intervals at which $g(x)$ is concave up or down, and inflection points.
e. Make a table consisting of all information above and then skech the graph of $g(x)$.

11. Determine the dimensions of **the rectangle of largest area** that can be inscribed in a **semicircle of radius** 3.

12. Evaluate the following integrals

$$(a). \int \frac{(\ln x)^2}{17x}dx \quad (b) \int \tan^3 x \sec x dx, \quad (c). \int \frac{x+2}{x^2+2}dx, \quad (d). \int \frac{x^2}{\sqrt{4x-x^2}}dx$$

$$(d). \int_2^5 \sqrt{1+x^4}\, x^7 dx \quad (e) \int \frac{x^4}{x^4-1}dx, \quad (f). \int x^3 \ln x\, dx, \quad (g). \int \frac{\sqrt{x-1}}{x}dx$$

13. Determine whether the following improper integrals are convergent OR divergent:

$$(a) \int_0^1 \frac{1}{\sqrt{x}\,(1+\sqrt{x})}dx$$

$$(b) \int_1^\infty \frac{2-e^{-x}}{x^{5/2}}dx$$