

PRACTICE PROBLEMS FOR MIDTERM EXAM

- Consider the region bounded by $y = \ln x$, $x = e$ and $y = 0$.
 - Draw the bounded region and calculate its area.
 - Give an integral formula for the circumference of the bounded region.
 - Rotate the region about y -axis and obtain a solid of revolution. Give an integral formula for its volume by using **the method of disc**.
 - Rotate the region about x -axis and obtain a solid of revolution. Give an integral formula for its volume by using **the method of cylindrical shells**.
 - Rotate the region about x -axis and obtain a solid of revolution. Give an integral formula for its surface area.

- Find the length of the parametric curve defined by equations below **from** $t = \frac{\pi}{4}$ **to** $t = \frac{\pi}{2}$

$$\begin{cases} x(t) = \sin t \\ y(t) = t + \cos t \end{cases} .$$

- Graph the region that lies the outside of $x^2 + y^2 = 4$ and the inside of $r = 4 \sin \theta$. Calculate its area.
- Graph the region that lies the outside of $r = 3$ and the inside of $r = 2(1 + \sin \theta)$. Calculate its area.
- (6 pts) Calculate $\lim_{n \rightarrow \infty} a_n$ where

$$(a) a_n = \left(\frac{n+1}{n-1}\right)^n, \quad (b) a_n = \left(\frac{3}{n}\right)^{1/n}, \quad (c) a_n = \frac{\ln n}{\sqrt{n}}, \quad (d) a_n = \frac{n!}{(n+1)^n}.$$

- Find the sum of the series $\sum_{n=1}^{\infty} \left(\frac{2}{3^n} + \frac{2n}{(n+2)(n+3)} \right)$.

- Determine whether the series below is convergent or divergent?

$$(a) \sum_{n=2}^{\infty} \frac{(n-1)(n+2)}{n^2}, \quad (b) \sum_{n=1}^{\infty} \frac{(\log n)^3}{n}, \quad (c) \sum_{n=3}^{\infty} \frac{1}{n\sqrt{\ln(n)}}, \quad (d) \sum_{n=1}^{\infty} \frac{2^n}{1+3^n}.$$

- Determine whether the following series is absolute convergent or conditional convergent or divergent

$$(a) \sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n+1}}, \quad (b) \sum_{n=1}^{\infty} (-1)^n \frac{2^n n^n}{n!}.$$

- Find the radius of convergence and the interval of convergence of the series below

$$(a) \sum_{n=1}^{\infty} \frac{(x+3)^n}{(n+1)5^n}, \quad (b) \sum_{n=0}^{\infty} \frac{n(-1)^n(x+1)^n}{2^n(n+1)}.$$