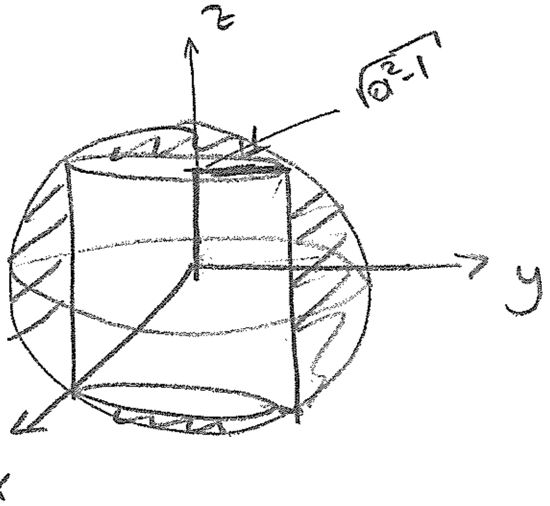
$\boxed{7.2.8}$ Ω_{xy}: $x^2+y^2+z^2=a^2$ nin içinde
$x^2+y^2=1$ in dışında



İki hacmin farkı olarak düşünerek ifade edelim. Küreden silindiri çıkaralım.

Kesişim noktaları
$z=\mp\sqrt{a^2-1}$

i) KARTEZYEN :

$$\underbrace{\int_{-a}^{a}\int_{-\sqrt{a^2-y^2}}^{\sqrt{a^2-y^2}}\int_{-\sqrt{a^2-x^2-y^2}}^{\sqrt{a^2-x^2-y^2}} dz\,dx\,dy}_{\text{küre}} - \underbrace{\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}}\int_{-\sqrt{a^2-1}}^{\sqrt{a^2-1}} dz\,dx\,dy}_{\text{silindir}}$$

ii) SİLİNDİRİK :

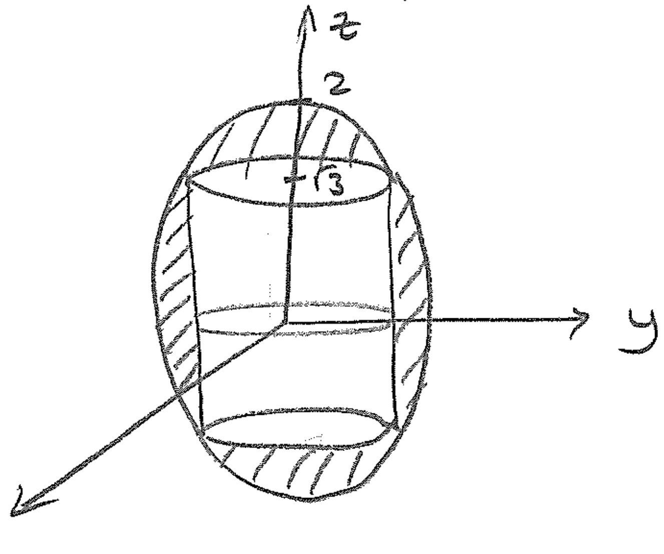
$$\int_{0}^{2\pi}\int_{0}^{a}\int_{-\sqrt{a^2-\rho^2}}^{\sqrt{a^2-r^2}} r.dz\,dr\,d\theta - \int_{0}^{2\pi}\int_{0}^{1}\int_{\sqrt{a^2-1}}^{\sqrt{a^2-1}} r.dz\,dr\,d\theta$$

iii) KÜRESEL :

$$\int_{0}^{2\pi}\int_{0}^{\pi}\int_{0}^{a}\rho^2\sin\phi\,d\rho\,d\phi\,d\theta - 2.\left[\int_{0}^{2\pi}\int_{0}^{\arctan\frac{1}{\sqrt{a^2-1}}}\int_{0}^{\sqrt{a^2-1}/\cos\phi}\rho^2\sin\phi\,d\rho\,d\phi\,d\theta + \int_{0}^{2\pi}\int_{\arctan\frac{1}{\sqrt{a^2-1}}}^{\pi/2}\int_{0}^{1/\sin\phi}\rho^2\sin\phi\,d\rho\,d\phi\,d\theta\right.$$

7.2.9

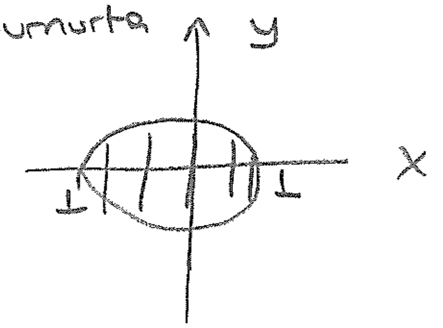
$$\Omega_{xyz} = \left\{ (x,y,z) : \ x^2+y^2 \geq \frac{1}{4} \ , \ 4x^2+4y^2+z^2 \leq 4 \right\}$$

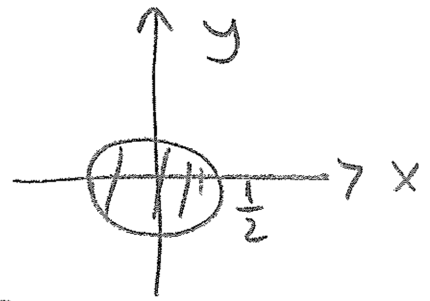


Yumurta ve silindir arasındaki bölge. En kolay olarak ikisinin hacmi farkı olarak ifade ederiz. Öyleyse

$$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{-\sqrt{4-4x^2-4y^2}}^{\sqrt{4-4x^2-4y^2}} \sqrt{x^2+y^2}\, dz\, dy\, dx \ - \underbrace{\int_{-\frac{1}{2}}^{\frac{1}{2}}\int_{-\sqrt{\frac{1}{4}-x^2}}^{\sqrt{\frac{1}{4}-x^2}}\int_{-\sqrt{3}}^{\sqrt{3}} \sqrt{x^2+y^2}\, dz\, dy\, dx}_{\text{silindir}}$$

Yumurta





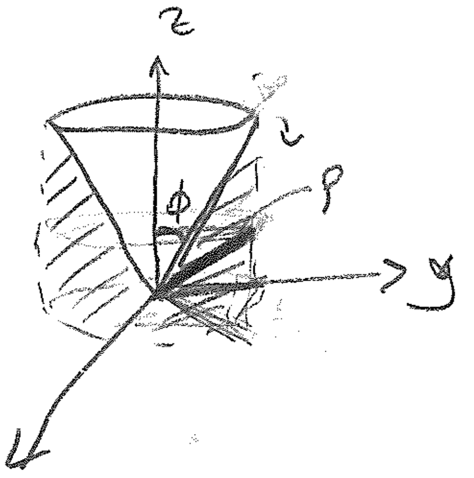
xy düzlemlerine izdüşüm

$\boxed{7.2.20}$

$$\int_{-3}^{3}\int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}}\int_{0}^{\sqrt{x^2+y^2}} x\,dz\,dx\,dy$$

integralini küresel koordinatlara dönüştürelim.

! $z=\sqrt{x^2+y^2}$ bize bir <u>koni</u> verir. (paraboloid değil.)



→ integral ile tanımlı bölge koninin altında kalan kısım.

Bölge, $x^2+y^2=9$ çemberlerinde taranıyor.

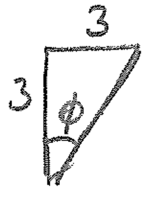
$$\Rightarrow \rho^2\sin^2\phi\cos^2\theta+\rho^2\sin^2\phi\sin^2\theta=9$$

$$\Rightarrow \rho^2\sin^2\phi=9$$

$$\Rightarrow \rho=\frac{3}{\sin\phi}$$ $\Longrightarrow$ öyleyse $\boxed{0<\rho<\frac{3}{\sin\phi}}$

ϕ açısı ise koniden ~~x~~y düzlemine kadar tarıyor.

Öyleyse



$\Longrightarrow \phi=\frac{\pi}{4}$ Yani $\frac{\pi}{4}\le\phi\le\frac{\pi}{2}$

θ açısı ise çemberi tarıyor $0\le\theta\le 2\pi$

$$\int_{0}^{2\pi}\int_{\pi/4}^{\pi/2}\int_{0}^{3/\sin\phi}(\rho\sin\phi\cos\theta)\,\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$