UYGULAMA

HATIRLATMALAR

→ f fonk R dikdörtgeni üzerindeki çift katlı integrali (limit varsa;

$$\iint_R f(x,y)\,\underbrace{dA}_{dxdy} = \lim_{m,n\to 0} \sum_{j=1}^{m}\sum_{i=1}^{n} f(x_i^*, y_j^*)\,\Delta A$$ ile tanımlıdır.

→ f fonksiyonu $R=[a,b]\times[c,d]$ o.ü

- R'de sınırlı
- E kümesi hariç R'de sürekli
- $\forall x\in[a,b]$ için $f(x,y)$ $[c,d]$ de integrallenebilir

⇒

$$\boxed{\iint_R f(x,y)\,dA = \int_a^b\int_c^d f(x,y)\,dy\,dx}$$

① Aşağıdaki integrallerin integrasyon bölgesini çizip, integrasyon sırasını değiştiriniz.

a) $\displaystyle\int_0^{\sqrt{2}}\int_{\frac{y^2}{2}}^{\sqrt{3-y^2}} f(x,y)\,dx\,dy$

b) $\displaystyle\int_{-2}^{1}\int_{x^2+4x}^{3x+2} dy\,dx$

c) $\displaystyle\int_0^{1}\int_{\frac{1-x^2}{2}}^{\sqrt{1-x^2}} f(x,y)\,dy\,dx$

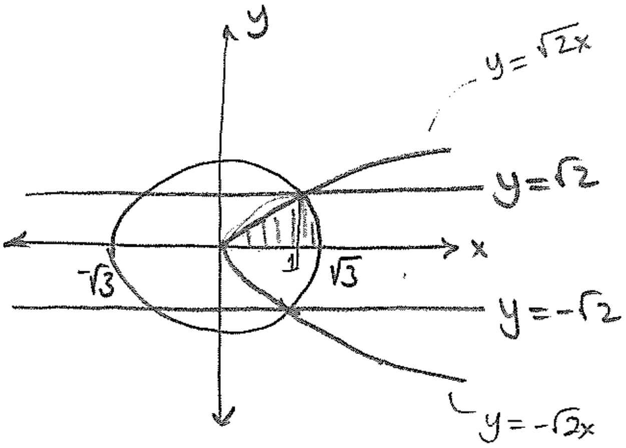
d) $\displaystyle\int_1^{2}\int_0^{\sqrt{4x-x^2}} f(x,y)\,dy\,dx$

a) $\underbrace{\frac{y^2}{2}}\le x \le \underbrace{\sqrt{3-y^2}}$

$x=\frac{y^2}{2}$ ve $x^2+y^2=3$ ortak çözersek: $\frac{y^4}{4}+y^2=3$

$(y^2+6)(y^2-2)=0$

$y=\mp\sqrt{2}$



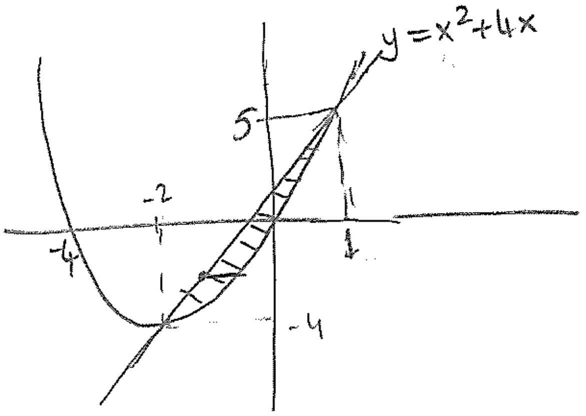
$$I=\int_0^1\int_0^{\sqrt{2x}} f(x,y)\,dy\,dx+\int_1^{\sqrt{3}}\int_0^{\sqrt{3-x^2}} f(x,y)\,dy\,dx$$

b) $\underbrace{x^2+4x}\le y\le \underbrace{3x+2}$

$y=x^2+4x$ ve $y=3x+2$ ortak çözümden $x^2+x-2=0$

$(x+2)(x-1)=0$

$x=-2$ veya $x=1$ olur.



$$I=\int_{-2}^{1}\int_{\frac{y-2}{3}}^{-2+\sqrt{4+y}} dx\,dy$$

$4+y=x^2+4x+4$

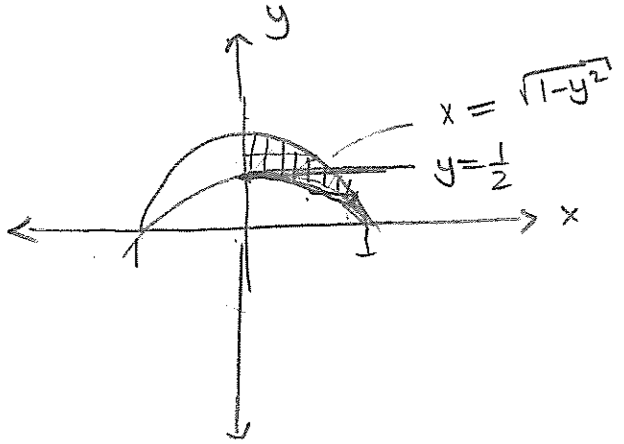
$(x+2)^2=4+y$

$x=-2\mp\sqrt{4+y}$

c) $\underbrace{\frac{1-x^2}{2}}_{} \le y \le \underbrace{\sqrt{1-x^2}}_{}$

$y=\frac{1-x^2}{2}$ ve $y^2+x^2=1$ ortak çözümünden $y^2+1-2y=1$

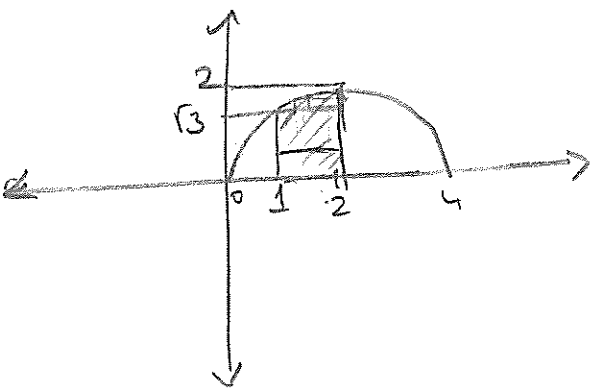
$y=0$ $\cancel{y=2}$



$$I=\int_0^{1/2}\int_{\sqrt{1-2y}}^{\sqrt{1-y^2}} f(x,y)\,dx\,dy+\int_{\frac{1}{2}}^{1}\int_0^{\sqrt{1-y^2}} f(x,y)\,dx\,dy$$

d) $\underbrace{0 \le y}_{} \le \underbrace{\sqrt{4x-x^2}}_{}$

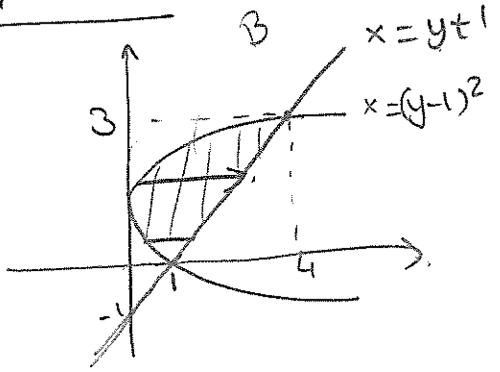
$y=0$ ve $(x-2)^2+y^2=4 \rightarrow x=-\sqrt{4-y^2}+2$



$$I=\int_1^{\sqrt{3}}\int_1^{2} f(x,y)\,dx\,dy+\int_{\sqrt{3}}^{2}\int_{-\sqrt{4-y^2}+2}^{2} f(x,y)\,dx\,dy$$

② $x=(y-1)^2$ eğrisi ile $y=x-1$ doğrusu arasındaki bölgenin alanı?

Çözüm:

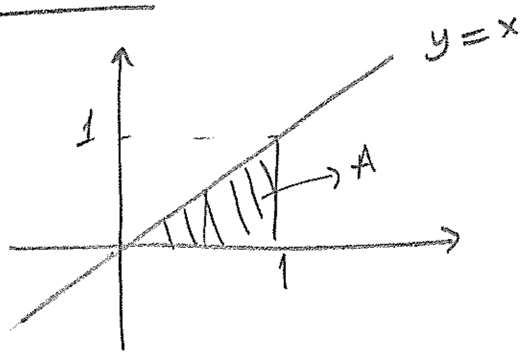


$$A=\iint_B dy\,dx$$

$$=\int_0^3\int_{(y-1)^2}^{1+y} dy\,dx=\frac{9}{2}\ \text{br}^2$$

③ $z = e^{x^2/2} + e^{y-y^2/2}$ fonk. altında ve $0 \le y \le x$ ve $0 \le x \le 1$ bölgesinin üzerinde kalan hacmi hesaplayınız.

Çözüm :



$$H = \iint_A f(x,y)\,dA$$

$$= \int_0^1 \int_0^x e^{\frac{x^2}{2}} + e^{y-\frac{y^2}{2}}\, dy\,dx$$

$$\Rightarrow \quad H = \int_0^1 \int_0^x e^{\frac{x^2}{2}}\, dy\,dx + \int_0^1 \int_0^x e^{y-\frac{y^2}{2}}\, dy\,dx$$

$$= \int_0^1 e^{\frac{x^2}{2}} . x\, dx + \int_0^1 \int_y^1 e^{y-\frac{y^2}{2}}\, dx\, dy$$

$$= \int_0^{1/2} e^w\, dw + \int_0^1 (1-y)\, e^{y-\frac{y^2}{2}}\, dy$$

$$= \left(e^{1/2} - 1\right) . 2$$

④ As. ardışık integralleri hesaplayınız.

a) $\int_{1}^{3}\int_{\frac{\pi}{6}}^{y^2} 2y\cos x \, dx\, dy$ b) $\int_{0}^{\pi}\int_{0}^{\sin x} y\, dy\, dx$

Çözüm:

a) $\int_{1}^{3}\int_{\frac{\pi}{6}}^{y^2} 2y\cos x\, dx\, dy = \int_{1}^{3} 2y\sin x \Big]_{\frac{\pi}{6}}^{y^2} dy = \int_{1}^{3} 2y\left(\sin y^2 - \sin\frac{\pi}{6}\right) dy$

$= -\int_{\cos 1}^{\cos 3} du - \sin\frac{\pi}{6}\, y^2\Big]_{1}^{3} = -\cos 3 + \cos 1 - 4$

$\cos y^2 = u$

$-\sin y^2\, 2y = du$

b) $\int_{0}^{\pi}\int_{0}^{\sin x} y\, dy\, dx = \int_{0}^{\pi} \frac{y^2}{2}\Big]_{0}^{\sin x} dx = \int_{0}^{\pi} \frac{\sin^2 x}{2}\, dx$

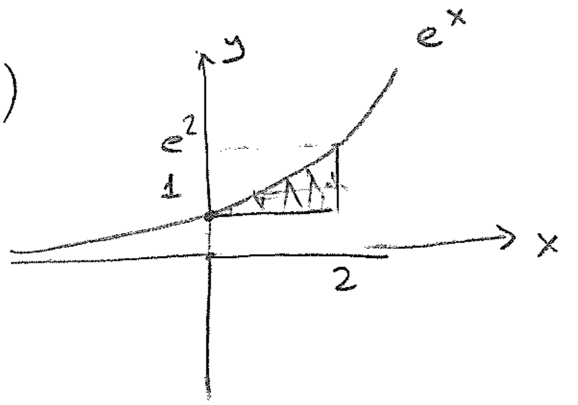
$= \int_{0}^{\pi} \frac{1-\cos 2x}{4}\, dx = \frac{\pi}{4}$

(5) Aş. integraller için integral bölgesini çizerek, integral sırasını değiştirip hesaplayınız.

a) $\int_0^2 \int_1^{e^x} dy\,dx$ b) $\int_1^{e^2} \int_{\ln y}^{2} dx\,dy$

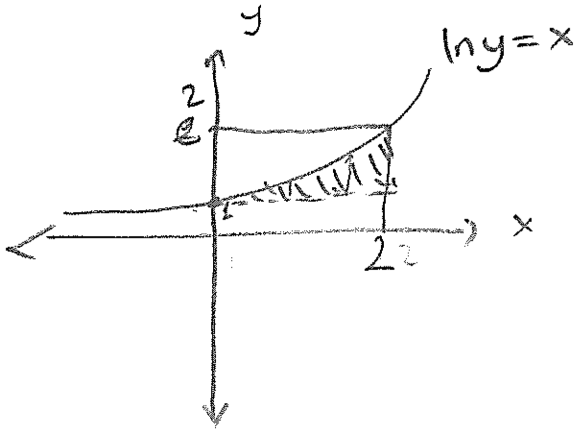
Çözüm;

a)



$$I = \int_1^{e^2} \int_{e^x}^{2} dx\,dy$$

b)



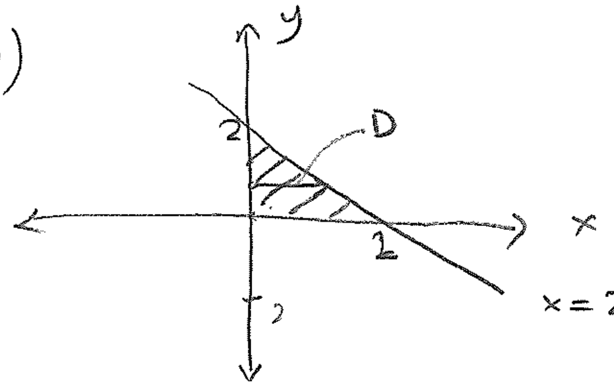
$$I = \int_1^{2} \int_1^{e^x} dy\,dx$$

⑥ a) $\iint_D (x+y)\,dA$ D: $x=0$, $y=0$ ve $x+y=2$

b) $\iint_D (x+y)\,dA$ D: $x^2+y^2=25$, $3x^2+16y=0$ ve $y\leq 0$

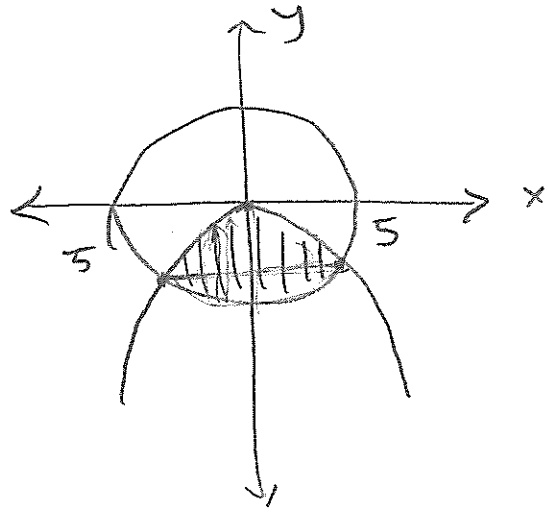
Çözüm:

a)



$$\int_0^2 \int_0^{2-y} (x+y)\,dx\,dy$$

b)



$x^2+y^2=25 \rightarrow y=\mp\sqrt{25-x^2}$

$-\frac{16y}{3}+y^2=25$

$3y^2-16y-75=0$

$3y \quad -25$

$y \quad +3$

$\boxed{y=-3}$ $\boxed{y=\frac{25}{3}}$

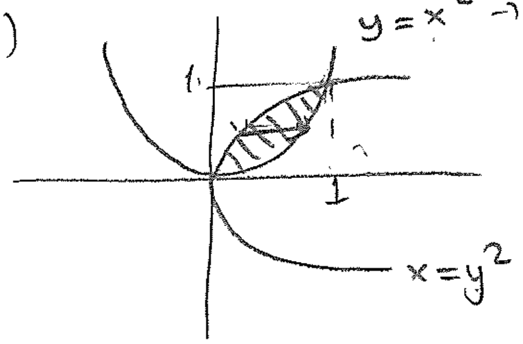
$$\int_{-3}^{\frac{25}{3}} \int_{-\sqrt{25-x^2}}^{\frac{-3x^2}{16}} (x+y)\,dy\,dx = \int\int (x+y)\,dx\,dy$$

⑦ a) $z=0$, $y=x^2$ ve $x=y^2$ ile sınırlanan $z=xy$ bölgesinin hacmi

b) $x=0$, $x=2$ ve $y=0$, $y=1$, $z=0$ ile sınırlanan $z=4-x^2$ hacmi

Çözümü

a)

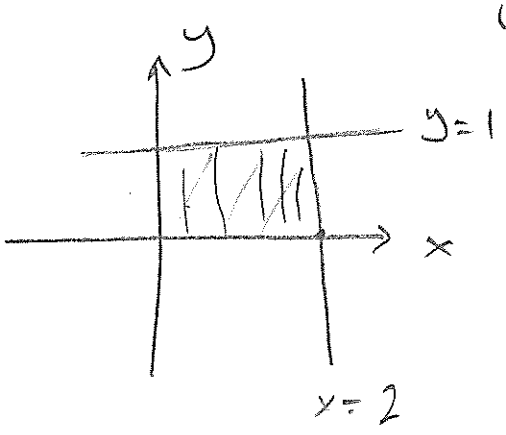


$y = y^4 \Rightarrow y=0 \quad y=1$

$y \langle 1$

$$\int_0^1 \int_{y^2}^{\sqrt{y}} xy \, dx \, dy$$

b)



$4-x^2=0$

$x^2=4$

$$\int_0^2 \int_0^1 4-x^2 \, dx \, dy$$

$$\iint_{D_{xy}} f(x,y)\,dA = \iint_{D_{r\theta}} f(r\cos\theta, r\sin\theta)\, r\,dr\,d\theta$$

Kutupsal Koordinatlara Geçiş,

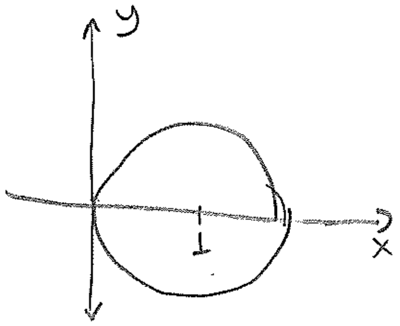
(8) $D_{xy}: x^2+y^2-2x=0$ çemberi ile sınırlanan bölge ö. kutupsal koordinatlara geçerek

$$\iint_{D_{xy}} \sqrt{x^2+y^2}\,dA \quad \text{integralini hesaplayınız}$$

Çözüm :

$$x^2+y^2-2x=0 \Rightarrow r^2-2r\cos\theta=0$$

$$\boxed{r=2\cos\theta}$$

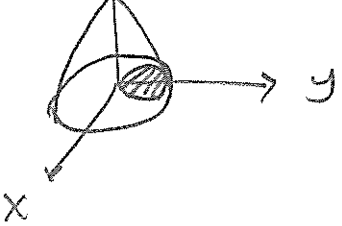


$$\iint_{D_{xy}} \sqrt{x^2+y^2}\,dA = \int_0^{\pi/2}\int_0^{2\cos\theta} r.r\,dr.d\theta$$

$$= \frac{32}{9} \;//$$

(9) Alttan xy-düzlemi üstten ve yandan $z=4-x^2-y^2$ yüzeyi ile sınırlanan ve $r=2\sin\theta$ silindiri içindeki bölgenin hacmi?

Çözüm



$$\int_0^{\pi}\int_0^{2\sin\theta} (4-r^2)\, r.\,dr\,d\theta$$